Addressing Inequality: Pathways to Inclusive Social Development

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Fifth Regional Seminar on Social Development

ECLAC – Santiago, Chile

24 June 2025

Introduction

Task: Provide insights on the measurement of **multidimensional inequality** and its policy linkages to **inclusive social development**

Social development

- Go **beyond GDP** and monetary measures: **Multidimensional**
- **People** based: Coherent **decompositions** and re-compositions

Divide into several questions

- Which **policies** work best?
- Which **dimensions** should be included?
- Which **types of inequality** should be prioritized?
- Why is there less clarity in policy directions with inequality than **poverty**?
- How should **multidimensional inequality** be measured?

Objective of this Presentation

Describe a new method of understanding multidimensional inequality that:

- Is **simple** and **grounded** in well-known technologies
- Adheres to the **axioms** of multidimensional inequality, including sensitivity to overlaps
- Can incorporate **policymakers**' views on the relative importance of dimension-specific inequalities
- Is appropriate for **hi-growth** environments
- Suggests new avenues for **policy**

Question: How to build such an index?

Intentional Measurement Framework

Foster 2025, Alkire, et al, Szekely, Lopez-Calva

To create measurement tools that lead to action

Before proceeding, identify:

Purpose, Concept, Desiderata, Axioms

Purpose

To monitor economic inequality in a country or region across time

Desiderata

*It must be **understandable** and easy to describe

It must conform to a **common-sense** notion of inequality

It must fit the **purpose** for which it is being developed

*It must be **technically** solid

It must be **operationally** viable

It must be easily **replicable**

First Key Desideratum

"**Understandable** and easy to describe"

Interpretation 1. **Intuitive**.

- Simple **structure** or functional form
- Like an "**Occam's** razor of measurement" (Simpler structure better than complex)

Interpretation 2. **Grounded**.

- **Linked** to well-understood elements
- Like **specific** inequalities or a measure of **association**

Which indices are **intuitive** and **grounded**?

Dashboard of Specific Inequalities

Pick **unidimensional** measure I (or the Lorenz curve L)

- Satisfying unidimensional **axioms**

Create a **dashboard** of domain-specific inequality levels

- A **vector** $(I_1, \ldots, I_d)$ or series of **Lorenz curves** $(L_1, \ldots, L_d)$
  - Inequality in **health**, inequality in **income**, etc.
- Called **specific inequalities** after Tobin's 1970 "specific egalitarianisms"

Example **EU** Multidimensional Inequality Monitoring Framework

- **Gini** in three domains $(G(x_{\cdot 1}), G(x_{\cdot 2}), G(x_{\cdot 3}))$

	Health	Income	Environment
Greece	0.08	0.32	0.09

Dashboard Pros and Cons

Pros

- Useful for documenting separate **specific** inequalities, and their trends
- Vector dominance of dashboards is **intuitive**; would satisfy many **axioms**

Cons

- No overall **headline** figure for multidimensional inequality
  - Arguably needed for **policy** salience
- No guidance when dimensions **disagree**
- Ignores **joint distribution** and positive association across dimensions

From dashboard to headline…

Average of Specific Inequalities

Take a **weighted average** of dashboard entries $A = \sum \omega_j I_j$

Where **weights** ω_j reflect relative importance of dimensions

Examples: Gajdos and Weymark (2005), Koshevoy and Mosler (1997)

Gini	H	I	E	Equal wts	Income 1/2
Greece	0.08	0.32	0.09	$A = 0.163$	$A' = 0.203$

Uses "Gini points" as common **measuring rod** with weights

Pros

- Clear **headline** figure
- Grounded in **specific** inequalities
- Would satisfy many **axioms**

Con Ignores **joint distribution** and positive association across dimensions

Q: Why is that important?

Example

	Situation A		Situation B	
	Dim 1	Dim 2	Dim 1	Dim 2
Person 1	1	1	1	0
Person 2	0	0	0	1
Dim Specific Ineq	Max	Max	Max	Max
Overall Inequality	Max		Less than Max	

Dimension specific inequalities are the same, and hence mislead

Overall structurally **different**: Situation A is fully unequal; Situation B is less so

Policy responses **different**: In Situation B, transfer could increase inequality

Need a **multidimensional** approach incorporating **association** across dimensions

Q/What tools are available to evaluate **multidimensional** inequality?

Second Key Desideratum

"It must be **technically** solid"

Interpretation. **Satisfies** accepted axioms.

Note: I see **axioms** as little **nuggets of policy**

Two main categories

- **Invariance** axioms: Which basic transforms leave measure unchanged?
- **Dominance** axioms: Which basic transforms can change the measure and how?

Recall: Core axioms for **unidimensional** inequality measure I:

- *Anonymity, Replication Invariance, Scale Invariance,* ***Transfer***

Core axioms for **multidimensional** inequality measure M:

- *Anonymity, Replication Invariance, Scale Invariance,* ***Uniform Majorization, Unfair Rearrangement***

Which indices?

Multidimensional Inequality Measures

There is a large **theoretical** literature

- Many **functional forms** for indices
- Satisfying a range of desirable **axioms**
  - Surveys: Aaberge and Brandolini 2015, Decancq and Lugo 2012, Seth and Santos 2018

Empirical literature much smaller

- Survey by Glassman 2019 lists 10 papers
- Often based on Maasoumi's 1986 two-stage approach

Policy impact smaller still

Why the lack of **take-up**?

Practical Barriers

Multidimensional inequality indices are "**intricate**"

- **Difficult** to explain to a non-specialist
- Hard to interpret what is actually being **measured**
- Unclear links to **specific** inequalities, positive **association**, or **growth** in a variable

In other words, they tend to **violate** Desideratum 1

Hopeless?

Not So Hopeless

Example: Multidimensional Poverty Index (**MPI**)

- Initially had **similar** barriers

Alkire and Foster 2011 *J. of Public E.*

- Strong **academic** uptake (5K+)
- Many **theoretical** and empirical pieces See OPHI at https://ophi.org.uk
- Significant **policy** impact See MPPN at https://www.mppn.org

Key applications

- **Global poverty:** UN's **Global MPI** including 110 countries, WB's MPM with 121
- **Regional poverty:** UN ESCWA's **Arab MPI** including 11 countries
- **National poverty:** 40-50 **National MPIs**, most recently **India** and **Nigeria**
- **Private sector:** **Citi** proposal for ESG investing

How was this done?

Barriers Removed

MPI has a **clear** structure

- **Easily explained** to non-specialist
- Simple weighted **counting** formula across dimensions
  - **Neutral** functional form - neither complements nor substitutes
- Grounded: Linked to unidimensional **FGT** measures Foster-Greer-Thorbecke (1984)

MPI satisfies axioms that support policy analysis

- **Decomposition** by subgroups
- Breakdown by dimension

Guided by **Intentional Measurement!**

Can this be done for multidimensional **inequality**?

Multidimensional and Specific Inequalities

Foster and Lokshin 2024 World Bank, ECA

Focus: Intuitive **Two-stage** Index Maasoumi 1986

Stage 1. Aggregate each person's achievements

Stage 2. Apply unidimensional inequality measure to resulting distribution

Characterization Result

Aggregation must be **linear** to satisfy core axioms of multidimensional inequality

"Technically solid"

Decomposition Result

Index can be expressed as **average specific inequalities** minus a term for association

"Understandability"

Calibration method

For selecting linear aggregate reflecting normative weights in base year

Illustrate with an examples

Data and Notation

Each person in a population has achievements in **several dimensions**

Example with 3 persons and 3 dimensions:

$$\begin{array}{lc} & \text{d many} \\ & \begin{array}{ccc} \text{Dim 1} & \text{Dim 2} & \text{Dim 3} \end{array} \\ n \text{ many} \quad \begin{array}{l} \text{Person 1} \\ \text{Person 2} \\ \text{Person 3} \end{array} & \begin{bmatrix} 8 & 2 & 4 \\ 3 & 3 & 3 \\ 1 & 1 & 8 \end{bmatrix} \end{array} \qquad x = [x_{ij}]$$

Multidimensional Inequality

$$M\begin{pmatrix} 8 & 2 & 4 \\ 3 & 3 & 3 \\ 1 & 1 & 8 \end{pmatrix}$$

Which index?

Specific Inequalities

$$I\begin{pmatrix} 8 \\ 3 \\ 1 \end{pmatrix}, \quad I\begin{pmatrix} 2 \\ 3 \\ 1 \end{pmatrix}, \text{ and } I\begin{pmatrix} 4 \\ 3 \\ 8 \end{pmatrix} \qquad I(x_{\cdot j})$$

Note: The ordering in each vector is irrelevant here

The Two-Stage Method

Constructs index $M(x)$ in **two stages** Maasoumi (1986)

Stage 1: Apply a function $h(x_{i1}, \dots, x_{id})$ = s_i to **aggregate** individual i's achievements

Stage 2: Apply **unidimensional** measure I to the distribution $s = (s_1, \dots, s_n)$

Multidimensional inequality index: $M(x) = I(s)$

Example

h is the geometric mean

I is the 2nd Theil measure

Both steps are **understandable** Many describe the method as “intuitive”

But does this yield a **coherent** multidimensional inequality index?

The Two-Stage Method

Original paper claimed M satisfies key multidimensional **transfer** axiom

Dardanoni (1995) gave **counterexample**

- Led some to dismiss **method**, others to question **axiom**
- **Empirical** applications continued using

Q: Is this violation really a problem?

Recall desideratum: “Index must be **technically solid**”

A: Yes. Must satisfy **core axioms** of multidimensional inequality indices

What are these axioms?

Invariance Axioms

Anonymity If x' is obtained from x by a **permutation** (of persons) then

$$M(x') = M(x)$$

Example

$$M\begin{pmatrix} 20 & 10 & 2 \\ 28 & 12 & 4 \\ 14 & 16 & 4 \end{pmatrix} = M\begin{pmatrix} 28 & 12 & 4 \\ 14 & 16 & 4 \\ 20 & 10 & 2 \end{pmatrix}$$

Invariance Axioms

Replication Invariance If x' is obtained from x by a **replication** then

$$M(x') = M(x)$$

Example

$$M\begin{pmatrix} 28 & 12 & 4 \\ 28 & 12 & 4 \\ 14 & 16 & 4 \\ 14 & 16 & 4 \\ 20 & 10 & 2 \\ 20 & 10 & 2 \end{pmatrix} = M\begin{pmatrix} 28 & 12 & 4 \\ 14 & 16 & 4 \\ 20 & 10 & 2 \end{pmatrix}$$

Invariance Axioms

Scale Invariance If x' is obtained from x by a **scalar multiple** then

$$M(x') = M(x)$$

Example

$$M\begin{pmatrix} 14 & 6 & 2 \\ 7 & 8 & 2 \\ 10 & 5 & 1 \end{pmatrix} = M\begin{pmatrix} 28 & 12 & 4 \\ 14 & 16 & 4 \\ 20 & 10 & 2 \end{pmatrix}$$

Dominance Axioms: Smoothing Dimensions

Definition We say x' is obtained from x by a **uniform smoothing** if

$$x' = Bx$$

for some **bistochastic** matrix B.

Example

$$B = \begin{pmatrix} 0.5 & 0.5 & 0 \\ 0.5 & 0.5 & 0 \\ 0 & 0 & 1 \end{pmatrix} \text{ and } x = \begin{pmatrix} 8 & 2 & 4 \\ 2 & 2 & 2 \\ 1 & 1 & 8 \end{pmatrix} \text{ implies } x' = \begin{pmatrix} 5 & 2 & 3 \\ 5 & 2 & 3 \\ 1 & 1 & 8 \end{pmatrix}$$

Uniform smoothing: B creates **progressive transfers** in each dimension

A First Transfer Axiom

Weak Uniform Majorization Axiom If x' is obtained from x by a **uniform smoothing** then $M(x') \leq M(x)$

Example

$$M\begin{pmatrix} 5 & 2 & 3 \\ 5 & 2 & 3 \\ 1 & 1 & 8 \end{pmatrix} \leq M\begin{pmatrix} 8 & 2 & 4 \\ 2 & 2 & 2 \\ 1 & 1 & 8 \end{pmatrix}$$

Paper also has **strict** version giving conditions when "<"

Note

- Uniform majorization axioms are multidimensional versions of the **transfer** axiom
- **Fundamental** to notion of multidimensional inequality

If M violates, then **not** measuring multidimensional inequality

Dominance Axioms: Rearranging Dimensions

Definition We say x' is obtained from x by an **unfair rearrangement** if for all dimensions, x' reorders all achievements from highest to lowest. In other words, $x'$is the **completely aligned** version $\bar{\bar{x}}$ of x

Example

$$x = \begin{pmatrix} 8 & 2 & 4 \\ 2 & 2 & 2 \\ 1 & 1 & 8 \end{pmatrix} \quad \text{and} \quad x' = \begin{pmatrix} 8 & 2 & 8 \\ 2 & 2 & 4 \\ 1 & 1 & 2 \end{pmatrix}$$

If **undo** an unfair rearrangement from x' to x, then inequality is relaxed

A second form of **progressive transfer** where a vector dominant person switches achievements with a dominated person

A Second Transfer Axiom

Weak Unfair Rearrangement Axiom If x' is obtained from x by an **unfair rearrangement** then $M(x') \geq M(x)$

Example
$$M\begin{pmatrix} 8 & 2 & 8 \\ 2 & 2 & 4 \\ 1 & 1 & 2 \end{pmatrix} \geq M\begin{pmatrix} 8 & 2 & 4 \\ 2 & 2 & 2 \\ 1 & 1 & 8 \end{pmatrix}$$

Paper also has **strict** version giving conditions when ">"

Note

- Unfair rearrangement axioms are multidimensional versions of the **transfer** axiom
- **Fundamental** to notion of multidimensional inequality

If M violates, then **not** measuring multidimensional inequality

- Aaberge and Brandolini (2015): "single feature that distinguishes multidimensional from unidimensional analysis"

Making the Two-Stage Method Technically Solid

We make the following assumption on **components**

- I satisfies **core** axioms (Lorenz consistency)
- $\boldsymbol{h}$ is continuous, concave, linear homogenous, and strictly increasing (Bosmans, et al, 2015)

When does the two-stage method yield a **coherent** index?

Theorem. Let M be a two-stage measure with components h and I. If M satisfies **weak uniform majorization**, then there exists $c \gg 0$ such that $\boldsymbol{h}(\boldsymbol{v}) = \boldsymbol{c}\boldsymbol{v}$ for all v.

Interpretation. The two-stage approach is **intuitively** appealing; but to have any chance at being **technically solid** the aggregation function must take on a **linear** form.

Note. Some empirical papers use other functional forms for h, and claim they are measuring multidimensional inequality. They are **not**.

Characterizations

Notation. Let $\mathcal{L}$ denote the set of two-stage indices M having **linear** h

Which other **axioms** are satisfied by $M \epsilon \mathcal{L}$?

Theorem. Any two-stage index $M \epsilon \mathcal{L}$ satisfies **all** core axioms of multidimensional inequality indices

Interpretation. **Technically** solid.

Corollary. A two-stage index M satisfies all core axioms **if and only if** $M \epsilon \mathcal{L}$.

Hence core axioms **characterize** subclass $\mathcal{L}$ among all two-stage measures

Our Approach Pros and Cons

Proposal: Use $M(x)$ derived as follows

Stage 1: Select h$(x_i) = cx_i$ for some c $= (c_1, \dots, c_d)$

Stage 2: Apply $I(\cdot)$ or $L(\cdot)$ to the distribution s $= (s, \dots, s_n)$

Note: This case of **linear** h has not been emphasized in the literature

Pro: Relatively **easy to understand** and explain, yet offers **flexibility** via the entries *c*; **neutral** case (as with MPI)

Pro: Satisfies the **core axioms** for multidimensional inequality measures

Indeed, it is the **only way forward** for the intuitive two-stage approach

Con: If interpret h as **utility** some might find it questionable

Note. We are following Atkinson and Bourguignon (1982, p190) by "making **no use** of information on **individual** relative valuations" of dimensional variables, and instead are treating the function h as "a subject for **social** decision."

Pro: Multidimensional from Specific Inequalities

Index is **linked** to

Term 1. Average **specific** inequalities

Term 2. A “**mobility measure**” capturing info on **joint distribution** (Shorrocks 1978)

Example M_G where G is **Gini** coefficient

Term 1. $A_G\left(x\right) = w_1 G\left(x_{.1}\right) + \cdots + w_d G(x_{.d})$ where $w_j = \frac{c_j\mu_j}{c_1\mu_1+\cdots+c_d\mu_d}$.

Term 2. $m_G(x) = M_G(\bar{\bar{x}}) - M_G(x) \geq 0$ where $\bar{\bar{x}}$ is completely aligned version of x

Example: $m_G(x) = M_G\begin{bmatrix} 7 & 5 \\ 4 & 2 \\ 2 & 1 \end{bmatrix} - M_G\begin{bmatrix} 4 & 1 \\ 2 & 5 \\ 7 & 2 \end{bmatrix}$ pure effect of alignment on index

Theorem. For M_G associated with the Gini coefficient, we have

$$M_G(x) = A_G\left(x\right) - m_G(x) \text{ for } x \in X$$

Interpretation

Multidimensional inequality M_G is

An **average** of specific inequalities

Minus the **mobility** term m_G

reflecting the pure impact of realignment

Hence intuitively ”**grounded**”

As x evolves, change in multidimensional inequality can be broken down into

(1) the changes in the **marginal distributions** as measured by the specific inequalities,
(2) the changes in **weights** on specific inequalities which depend on the means of the marginal distributions, and

(3) the change in the **joint distribution** as measured by Shorrocks mobility.

Applying the Lorenz

Example $L_s(p)$ is the **Lorenz** curve of the aggregate vector s

Term 1. $L_A(p) = w_{\mathbf{1}} L_1(p) + \cdots + w_{\boldsymbol{d}} L_d(p)$ for $p \in [0,1]$

Term 2. $m_L(p) = L_s(p) - L_{\bar{\bar{s}}}(p) \geq 0$ where $\bar{\bar{s}}$ is the aggregate vector of $\bar{\bar{x}}$

Theorem. For the Lorenz curve implemented with $c \gg 0$, we have

$$L_s(p) = L_A(p) + m_L(p) \qquad \text{for } p \in [0,1]$$

Note. Integrating $m_L(p)$ measures the **area** between two Lorenz curves, and hence yields the mobility term for the **Gini** coefficient.

Calibrating the Index

How to **select** $c = (c_1, \dots, c_d)$?

Start with data in a **base** year

Any given *c* yields an associated **average** Lorenz curve

$$L_A(p) = w_1 L_1(p) + \cdots + w_d L_d(p)$$

Suppose policymaker has normative **values** for the specific inequalities

Example ½ on income inequality, ½ equally divided across remaining inequalities

The normative weights generate a **normative** average Lorenz curve

Now find coefficients *c* so that the two average Lorenz curves are the **same**

Solution: $\boldsymbol{c_j = \frac{w_j}{\mu_j^b}}$ ensures that $L_A(p)$ corresponds to the normative average Lorenz curve for the base year

Theory Summary

The index

- Has a simple **structure** or functional form
- Satisfies all **core axioms**
- Can be broken down into **specific inequalities** and a term for **association**.
- Can be **calibrated** to reflect normative weights associated with policy priorities

Next: Empirical illustrations

Empirical Illustrations

Azirbaijan: EBRD/WB Life in Transition Survey Round 3 (2016) and Round 4 (2023)

Impact of **rapid growth** on MDI

Albania: WB (2018)

Regional rankings of income inequality and MDI

Impact of a simulated increase in a public good

Simulated changes in mean, specific inequality, correlation

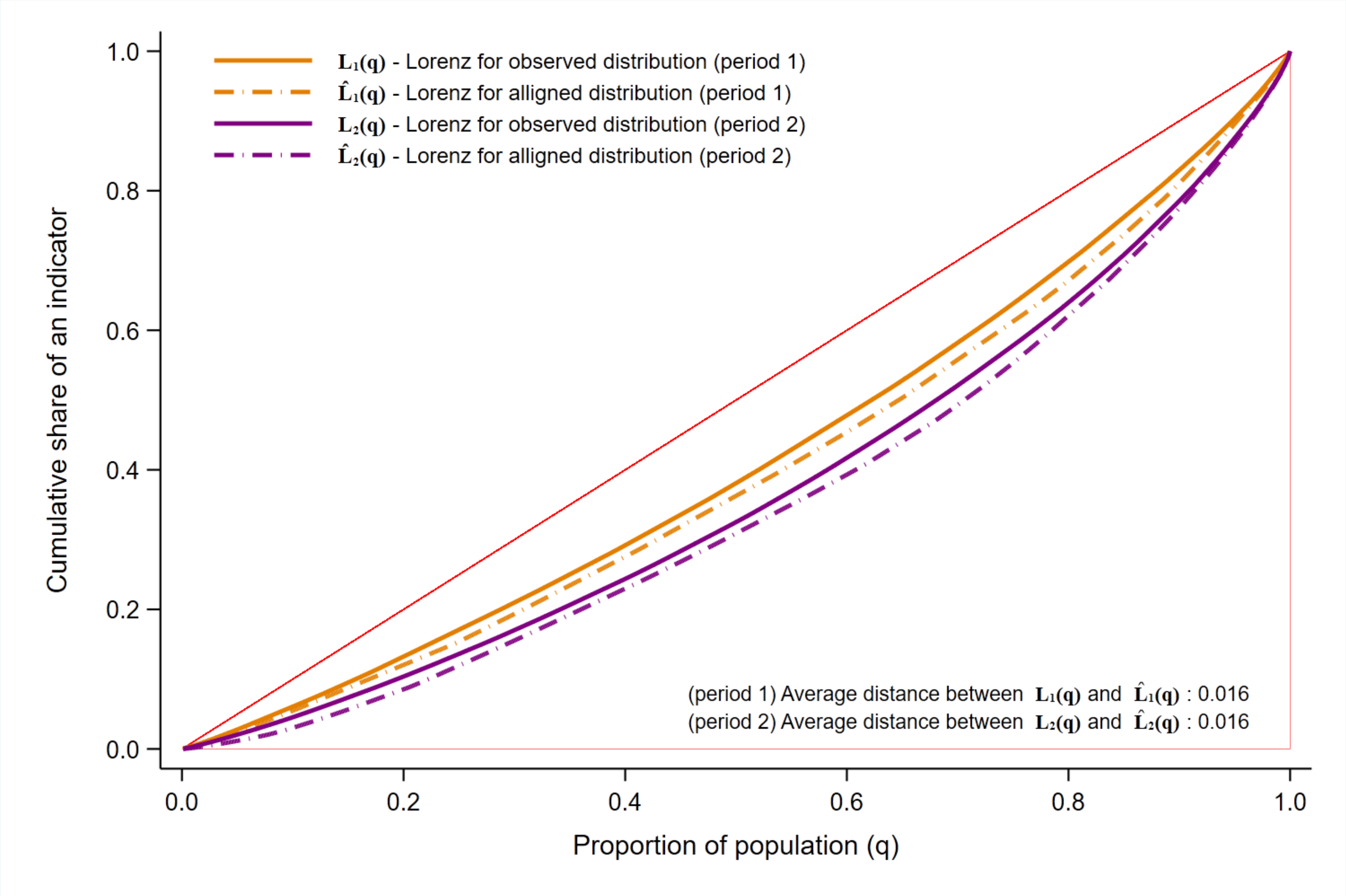
Our measure and others

Example: Change in Inequality in Azerbaijan

	2016	2023
Specific Inequalities Income		
Mean	852.46	1384.44
Gini	0.253	0.339
Education (years)		
Mean	10.304	11.091
Gini	0.094	0.115
Health		
Mean	3.448	3.511
Gini	0.166	0.159
Multidimensional Inequality		
$A(x) = M(\bar{\bar{x}})$	0.181	0.260
$M(x)$	0.144	0.230
$m(x) = R(x)$	0.037	0.029

EBRD/WB Life in Transition Survey Round 3 (2016) and Round 4 (2023)

Lorenz Curves for Azerbaijan 2016 and 2023

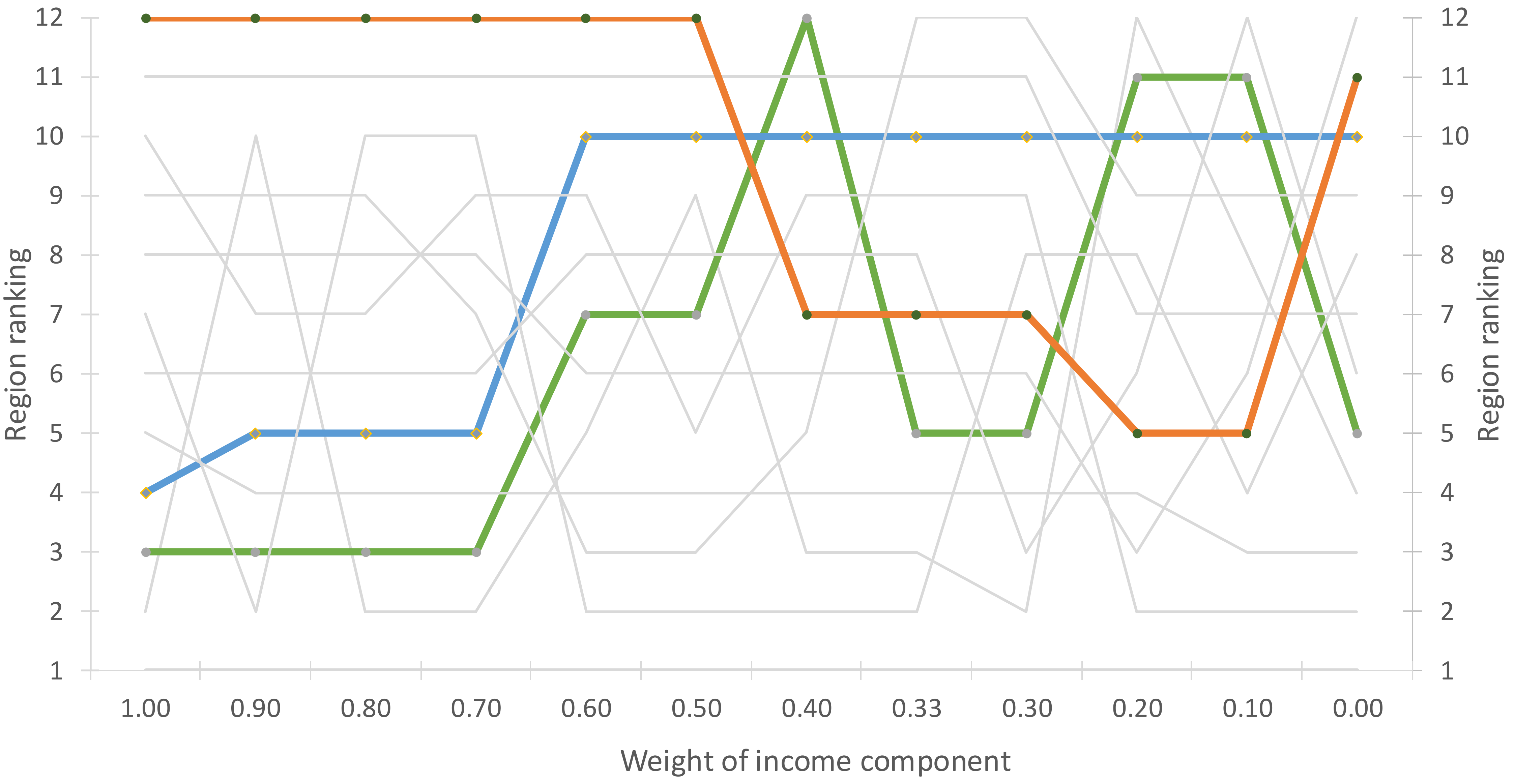


EBRD/WB Life in Transition Survey Round 3 (2016) and Round 4 (2023)

Inequality Ranking of Albania’s Regions in 2018

Albania regions	Education	Living space	Income	W. Average	MDI
Dibër	8	9	12	9	7
Vlorë	5	1	11	11	11
Fier	6	7	10	12	9
Durrës	10	2	9	7	12
Elbasan	1	10	8	5	6
Shkodër	7	6	7	3	2
Korçë	11	8	6	8	8
Gjirokastër	12	3	5	4	4
Tiranë	9	12	4	6	10
Lezhë	3	5	3	2	5
Kukës	2	4	2	10	3
Berat	4	11	1	1	1

Changes in Albania's Regional Ranking by Weights



Simulated Effect of a Uniform, 1-Year Increase in Education

Albania regions	EDU	Living Space	Income	W. AVG	MDI	Simulated +1 year of EDU		
						EDU	W.AVG	MDI
Dibër	8	9	12	9	7	8	9	7
Vlorë	5	1	11	11	11	4	1	11
Fier	6	7	10	12	**9**	6	12	**2**
Durrës	10	2	9	7	12	10	7	9
Elbasan	1	10	8	5	**6**	1	3	**12**
Shkodër	7	6	7	3	**2**	11	8	**8**
Korçë	11	8	6	8	8	7	5	6
Gjirokastër	12	3	5	4	4	12	4	3
Tiranë	9	12	4	6	10	9	6	10
Lezhë	3	5	3	2	5	3	2	5
Kukës	2	4	2	10	3	2	10	4
Berat	4	11	1	1	1	5	11	1

Changes in Multidimensional Inequality between Two Periods for Different Inequality Indicators and Three Simulation Scenarios.

	Mean (100 → 200)		Gini (0.3 → 0.4)		Correlation (0 → 0.3)	
	Period 1	Period 2	Period 1	Period 2	Period 1	Period 2
AKS α = -1; β = -1	0.454	0.453	0.454	0.519 ✓	0.454	0.506
MLD α = 1; β = 1	0.261	0.261	0.261	0.306 ✓	0.261	0.261
BGN α = -1; β = 0.5	0.131	0.104 ✓	0.131	0.164 ✓	0.131	0.107 ✓
BGN α = 0.5; β = 0.5	0.082	0.088	0.082	0.093 ✓	0.082	0.083
BGN α = 0.5; β = 0	0.107	0.107	0.107	0.128 ✓	0.107	0.096 ✓
FLS α = -1	0.379	0.331 ✓	0.379	0.476 ✓	0.379	0.379
FLS α = 0	0.261	0.261	0.261	0.306 ✓	0.261	0.261
MDI (Gini)	**0.400**	**0.434**	**0.400**	**0.450**	**0.400**	**0.422**
MDI (Theil)	**0.281**	**0.335**	**0.281**	**0.360**	**0.281**	**0.317**
MDI (MLD)	**0.270**	**0.323**	**0.270**	**0.356**	**0.270**	**0.278**

The simulated distributions have the following parameters: First outcome: mean 100, Gini 0.5; Second outcome: mean 30, Gini 0.3; Correlation: period 1: 0; period 2: ≈ 0.3.

Recap

Describe a new method of understanding multidimensional inequality that:

- Is **simple** and **grounded** in well-known technologies
- Adheres to the **axioms** of multidimensional inequality, including sensitivity to overlaps
- Can incorporate **policymakers**’ views on the relative importance of dimension-specific inequalities
- Is appropriate for **hi-growth** environments
- Suggests new avenues for **policy**

Question: What do you think?

Thank you!