Spatial Spillovers and Productive Efficiency: A Spatial Stochastic Frontier Model for Panel Data

André Luiz Ferreira e Silva   André Luis Squarize Chagas   Carlos Roberto Azzoni

Agenda

Introduction

- Regional efficiency are commonly associated with factor dotation and local technology
- Spatial frontier analysis explore the potential spillover of the inefficiency
- In the spatial analysis of production efficiency, spatial autocorrelation can be controlled through the SAR (spatial autoregressive) specification. Heterogeneity, in turn, depends on the hypotheses attributed to the stochastic frontier errors.
- Our proposal consists of integrating the SAR component into the structure of a stochastic frontier model with random effects.

Stochastic Frontier

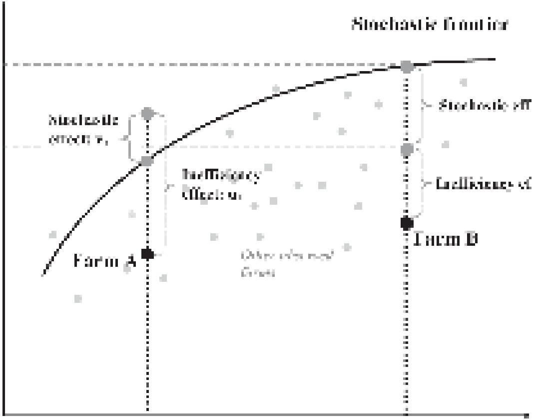


Figure: Stochastic frontier model (Coelli et al., 2005; Lakner et al., 2012)

Panel data stochastic frontier models

First generation of panel data stochastic frontier models (Pitt and Lee, 1981; Kumbhakar, 1987a,b; Battese and Coelli, 1988)

$$y_{it} = \alpha + f(\mathbf{x}_{it};\beta) + v_{it} - u_{it} \tag{1}$$

where

- deterministic factors: α, β's, $\mathbf{x}_{it}$ and the technology ($f(\cdot)$)
- random factors: v_{it} and u_{it}

Panel data stochastic frontier models

- Different specifications to u_{it} and v_{it} result in different models:
- Battese and Coelli (1992):

$$\begin{aligned} y_{it} &= \alpha + f(\mathbf{x}_{it};\beta) + v_{it} - u_{it}, \\ u_{it} &= \exp[(-\eta(t-T))]u_i, \quad u_i \overset{i.i.d.}{\sim} N^+(\mu, \sigma_u^2), \\ v_{it} &\overset{i.i.d.}{\sim} N(0, \sigma_v^2). \end{aligned} \tag{2}$$

- Battese and Coelli (1995):

$$u_{it} = z_{it}\delta + \omega_{it}, \quad u_{it} \overset{i.i.d.}{\sim} N^+(z_{it}\delta, \sigma_u^2) \tag{3}$$

where $\omega_{it} \geq -z_{it}\delta$.

Panel data stochastic frontier models

- Kumbhakar and Lovell (2003):

$$\begin{aligned} y_{it} &= \alpha + f(\mathbf{x}_{it};\beta) + v_{it} - u_{it} \\ u_{it} &\sim N^{+}(\mu,\sigma_u^2) = N^{+}(\mu,\exp(z_{u,it}\delta)) \\ v_{it} &\sim N(0,\sigma_v^2) = N(0,\exp(z_{v,it}\phi)). \end{aligned} \tag{4}$$

- Greene (2005a,b) proposed two models, which he called ‘true’ fixed-effects frontier model and ‘true’ random effects frontier model.

$$\begin{aligned} y_{it} &= \alpha_i + f(\mathbf{x}_{it};\beta) + v_{it} - u_{it} \\ u_{it} &\sim N^{+}(\mu,\sigma_u^2) = N^{+}(\mu,\exp(z_{u,it}\delta)) \\ v_{it} &\sim N(0,\sigma_v^2) = N(0,\exp(z_{v,it}\phi)) \\ \alpha_i &= N(0,\sigma_\alpha^2). \end{aligned} \tag{5}$$

Spatial autoregressive stochastic frontier models for panel data

- Glass et al. (2016):

$$\begin{aligned} y_{it} &= \rho \sum_{j \neq i} w_{ij} y_{jt} + \mathbf{x}_{it}^{\top} \beta + v_{it} - u_{it} \\ u_{it} &\sim N^{+}(0, \sigma_u^2) \\ v_{it} &\sim N(0, \sigma_v^2). \end{aligned} \tag{6}$$

- Lai and Tran (2022):

$$y_{it} = \rho \sum_{j \neq i} w_{ij} y_{jt} + \mathbf{x}_{it}^{\top} \beta + (\alpha_i - \eta_i) + (v_{it} - u_{it}) \tag{7}$$

where, $\xi_{1,i.} = (\alpha_i - \eta_i)\iota_T$ represents the time-invariant and $\xi_{2,i.} = (v_{it} - u_{it})$ time-variant random components.

Spatial autoregressive stochastic frontier models for panel data

- Ramajo and Hewings (2018):

$$\begin{aligned} y_{it} &= \rho \sum_{j \neq i} w_{ij} y_{jt} + f(\mathbf{x}_{it}; \beta) + v_{it} - u_{it}, \qquad (8) \\ u_{it} &= \exp[(-\eta(t-T))] u_i, \quad u_i \overset{i.i.d.}{\sim} N^+(\mu, \sigma_u^2), \\ v_{it} &\overset{i.i.d.}{\sim} N(0, \sigma_v^2). \end{aligned}$$

Spatial stochastic frontier model with random effects (SSF-RE)

Our proposal: Spatial Stochastic Frontier with Random Effects (SSF-RE).

$$y_{it} = \alpha_i + \rho \sum_{j \neq i} w_{ij} y_{jt} + \mathbf{x}_{it}^{\top} \beta + v_{it} - u_{it} \tag{9}$$

Let $y_{i.}$ be the column vector formed by stacking $y_{i1}, \ldots, y_{iT}$, $\mathbf{x}_{i.}$, $v_{i.}$, $u_{i.}$ and $\alpha_{i.} = \alpha_i \iota_T$ are defined similarly, where ι_T is a vector of ones. Then (9) in matrix form can be written as follows:

$$y_{i.} = \rho \sum_{j \neq i} w_{ij} y_{j.} + \mathbf{x}_{i.} \beta + \alpha_{i.} + \xi_{i.} \tag{10}$$

In this specification, $\xi_{i.} = v_{i.} - u_{i.}$ is the time-variant error term.

Assumptions of the random effects

We assume that specific random effects and time-varying error term are subject to the following assumptions:

A1: Unobserved individual specific effects remain normally distributed with zero mean and heteroscedastic variance, such that: $\alpha_i \sim \mathrm{N}(\mathbf{0}, \sigma_\alpha^2(\mathbf{z}_i))$, where $\sigma_\alpha^2(\mathbf{z}_i) = \exp(\delta^\top \mathbf{z}_i)$ for $\mathbf{z}_i \in \mathbb{R}^n$ and $\mathbf{z}_i$ are exogenous variables.

A2: The two time-varying random components have the following distribution: $v_{i.} \sim \mathrm{N}(\mathbf{0}_T, \sigma_v^2 \mathbf{I}_T)$ and $u_{i.} \sim \mathrm{N}^+(\mathbf{0}_T, \Sigma_i)$, where $\Sigma_i = \mathrm{diag}(\sigma_u^2(\mathbf{z}_{i1}), \ldots, \sigma_u^2(\mathbf{z}_{iT}))$ such that $\sigma_u^2(\mathbf{z}_{it}) = \exp(\phi^\top \mathbf{z}_{it})$ for $\mathbf{z}_{it} \in \mathbb{R}^n$. Here, $\mathbf{0}_T$ is a $(T \times 1)$ vector of zeros and $\mathbf{I}_T$ is a $(T \times T)$ identity matrix.

A3: The three random components α_i, v_{it} and u_{it} are independent to each other and uncorrelated with $\mathbf{x}_{it}$.

Closed-skew normal distribuition

Let $\phi_T(\cdot;\mu,\Xi)$ and $\Phi_T(\cdot;\mu,\Xi)$ be the marginal and cumulative normal probability density functions, Lai and Kumbhakar (2018) and Lai and Tran (2022) demonstrate that the time-variant component follows the closed skew normal distribution, according to the following parameters

$$\xi_{i.} \sim CSN_{T,T}\left(\mathbf{0}_T, \Theta_i, -\Sigma_i\Theta_i^{-1}, \mathbf{0}_T, \Upsilon_i\right), \tag{11}$$

where $\Theta_i = \sigma_v^2\mathbf{I}_T + \Sigma_i$ and $\Upsilon_i = (\frac{1}{\sigma_v^2}\mathbf{I}_T + \Sigma_i^{-1})^{-1}$.

The joint pdf of time-variant compound error term is given by

$$f_\xi(\xi_{i.}) = 2^T\phi_T(\xi_{i.};\mathbf{0}_T,\Theta_i)\,\Phi\left(-\Sigma_i\Theta_i^{-1}\xi_{i.};\mathbf{0}_T,\Upsilon_i\right), \tag{12}$$

Joint PDF and MSL estimator

Under the distribution assumptions of [A1]-[A3], the log-likelihood of the maximum simulated likelihood estimator can be obtained from the joint probability density function.

$$f_{\varepsilon}(\varepsilon_{i.}) = \int_{-\infty}^{+\infty} f_{\varepsilon|\alpha}(\varepsilon_{i.}|\alpha_{i.}) f_{\alpha}(\alpha_{i.}) d\alpha_{i.} = \int_{-\infty}^{+\infty} f_{\xi}(\varepsilon_{i.} - \alpha_{i.}) f_{\alpha}(\alpha_{i.}) d\alpha_{i.} \tag{13}$$

where $\varepsilon_{i.} = \alpha_{i.} + \xi_{i.}$ is the error term of the SSF-RE model.

Given that Θ_i and Υ_i are both diagonal matrices, the T-dimensional integration can be reduced to a one-dimensional integration. Thus, the joint pdf (13) can be further simplified as

$$f_{\xi}(\xi_{i.}) = 2^T \prod_{t=1}^{T} \left[\phi\left(\xi_{i.}; 0, \sigma_v^2 + \sigma_{u_{it}}^2\right) \Phi\left(\frac{-\sigma_{u_{it}}^2}{\sigma_v^2 + \sigma_{u_{it}}^2} \xi_{i.}; 0, \frac{\sigma_v^2 \sigma_{u_{it}}^2}{\sigma_v^2 + \sigma_{u_{it}}^2} \right) \right]. \tag{14}$$

Maximum Simulated Likelihood (MSL) estimator

Using hypothesis [A3], the conditional function in (13) can be replaced by (14), then obtain an approximate empirical model through simulation

$$f^s_\varepsilon(\varepsilon_{i.}) = \frac{1}{M}\sum_{m=1}^{M} f_\xi\left(\varepsilon_{i.} - \alpha^m_{i.}\right). \tag{15}$$

The joint function (15) controls the random effects not observed in the SSF model. The function $f_\xi(\cdot)$ receives a normally distributed random variable as a Halton draw.

Maximum Simulated Likelihood (MSL) estimator

Using (14)-(15) and defining $\theta = (\rho, \beta^\top, \sigma_v, \sigma_u, \sigma_\alpha)^\top$ as a vector of parameters belonging to the probability space Θ, it is possible to derive the log-likelihood function for SSF-RE model

$$\begin{aligned} LL^s(\theta) &= \sum_{i=1}^{N} \ln f_\varepsilon^s(\varepsilon_{i.}) \\ &= \sum_{i=1}^{N} \ln \left\{ \frac{1}{M} \sum_{m=1}^{M} \left[2^T \phi\left(\varepsilon_{i.} - \alpha_{i.}^m; 0, \sigma_v^2 + \sigma_{u_{it}}^2\right) \times \prod_{j=1}^{T} \Phi\left(\frac{-\sigma_{u_{it}}^2}{\sigma_v^2 + \sigma_{u_{it}}^2} \xi_{i.}; 0, \frac{\sigma_v^2 \sigma_{u_{it}}^2}{\sigma_v^2 + \sigma_{u_{it}}^2} \right) \right] \right\} \end{aligned} \quad (16)$$

The optimization of equation (16) results in the simulated maximum likelihood estimator.

$$\hat{\theta}_{SML} = \underset{\theta \in \Theta}{\operatorname{argmax}} \; LL^s(\theta) \quad (17)$$

Spatial prediction of efficiency and marginal effects

Let $y_{.t} = (y_{1t}, \ldots, y_{Nt})'$ be a $(N \times 1)$ vector formed by stacking the cross-sectional units; $\mathbf{x}_{.t}, \alpha, v_{.t}$ and $u_{.t}$ are defined in a similar way. Thus, (9) for the observations at time t can be written as follows:

$$y_{.t} = \rho \mathbf{W} y_{.t} + \mathbf{x}_{.t}\beta + \alpha + v_{.t} - u_{.t} \tag{18}$$

Given that $\mathbf{I}_N - \rho\mathbf{W}$ is non-singular, the SSF-RE model can be written to the reduced form

$$y_{.t} = \mathbf{S}(\rho)\mathbf{x}_{.t}\beta + \mathbf{S}(\rho)\alpha + \mathbf{S}(\rho)(v_{.t} - u_{.t}), \tag{19}$$

where $\mathbf{S}(\rho) = (\mathbf{I}_N - \rho\mathbf{W})^{-1}$ is a $(N \times N)$ matrix.

Let β_k be the k^{th} element of β and $S_{ij}(\rho) \in \mathbf{S}(\rho)$. Then, a change in the k^{th} explanatory variable from firm j at time t causes the following marginal effect (LeSage and Pace, 2009)

$$\frac{\partial y_{it}}{\partial x_{jt,k}} = \beta_k S_{ij}(\rho). \tag{20}$$

Spatial prediction of efficiency and marginal effects

Glass et al. (2016): by definition, the total impact is the sum of all spatial effects

$$\beta_k^{\mathrm{Tot}} = \sum_{j=1}^{N} \frac{\partial y_{it}}{\partial x_{jt,k}} = \beta_k \sum_{j=1}^{N} S_{ij}(\rho). \tag{21}$$

Lai and Tran (2022) argue that is only valid if the frontier is linear. When is nonlinear in x_{it}, these two equations need to be adjusted accordingly.

Spatial prediction of efficiency and marginal effects

Given the MSL estimator of θ, Kutlu (2018) and Lai and Tran (2022) decomposed the effects into two vectors

$$u_{it}^{\text{Tot}} = S_{ii}(\rho)u_{it} + \sum_{j\neq i}^{N} S_{ij}(\rho)u_{jt} \tag{22}$$

where the terms on the right-hand side represent the direct and indirect impacts of the transient inefficiency.

Using a multiplicative formulation, we propose decompose the total impact of the efficiency of local firm i as

$$\begin{aligned} \text{TE}_{it}^{\text{Tot}} &= \exp\left(-S_{ii}(\rho)u_{it}\right) \times \prod_{j\neq i}^{T} \exp\left(-S_{ij}(\rho)u_{jt}\right) \\ &= \text{TE}_{it}^{\text{Dir}} \times \text{TE}_{jt}^{\text{Ind}} \end{aligned} \tag{23}$$

Spatial prediction of efficiency and marginal effects

There are two ways to compute transient inefficiency.

Filippini and Greene (2016) used the moment generating function to predict technical efficiency.

Lai and Tran (2022) argue that this method can be very empirically exhaustive due to the multivariate derivation of the cumulative pdf.

Based on the law of iterated expectation (LIE) of Bayes, they suggest a simpler approach to estimating inefficiency (efficiency) and marginal effects.

$$\mathbb{E}^s(u_{it}|\varepsilon_{i.}) = \frac{1}{M}\sum_{m=1}^{M}\left\{\tilde{\mu}_{it}^m + \tilde{\sigma}_{it}\left[\frac{\phi(-\tilde{\mu}_{it}^m/\tilde{\sigma}_{it})}{1-\Phi(-\tilde{\mu}_{it}^m/\tilde{\sigma}_{it})}\right]\right\}\Omega_i^m, \tag{24}$$

where $\tilde{\sigma}_{it}^2 = \frac{\sigma_v^2\sigma_{u_{it}}^2}{\sigma_v^2+\sigma_{u_{it}}^2}$, $\tilde{\mu}_{it}^m = \frac{-\xi_{i.}\sigma_{u_{it}}^2}{\sigma_v^2+\sigma_{u_{it}}^2}$ and $\Omega_i^m = \frac{f_\xi(\varepsilon_{i.}-\alpha_{i.}^m)}{\frac{1}{M}\sum_{m=1}^{M} f_\xi(\varepsilon_{i.}-\alpha_{i.}^m)}$.

Conditional expectation (24) returns a vector of transient inefficiency which is used in the spatial prediction of the firm's efficiency.

Data

- We used data of the Brazilian food industry.
- It is the industry least geographically concentrated and has expanded rapidly.
- Food industry increased its participation in manufacturing employment from 13.4% in 2007 to 18% in 2018
- In terms of VA, it was 18.17% in 2007 jumped to 24.4% in 2018.

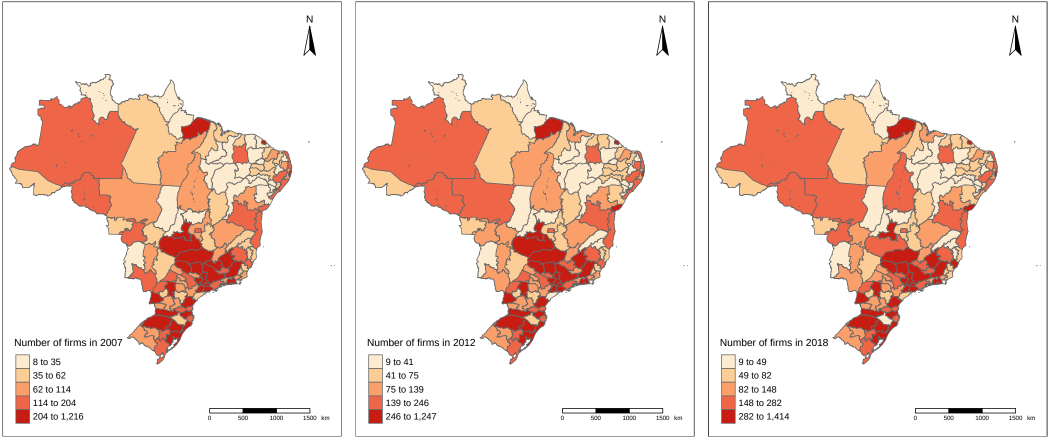
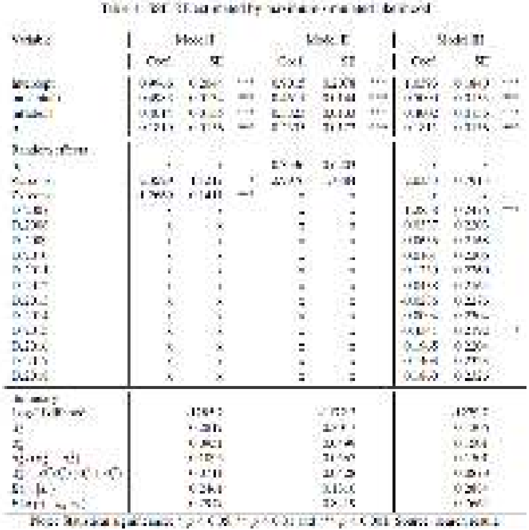


Figure: Regional distribution of food industry in Brazil



Density of technical efficiency

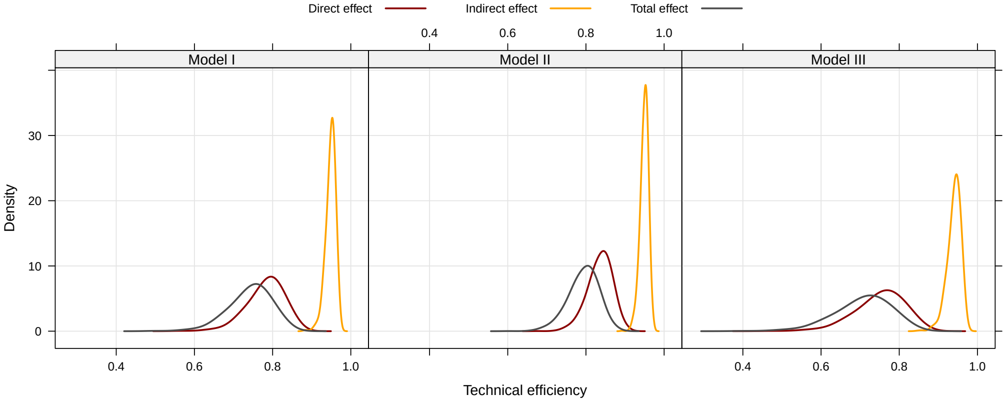
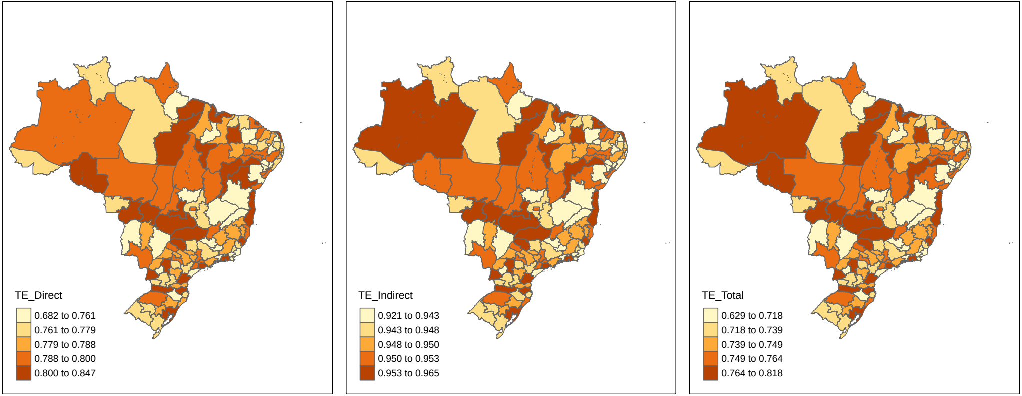


Figure: Density of technical efficiency: direct, indirect and total effect.

Model I – Spatial distribution of technical efficiency

Figure: I - Spatial distribution of technical efficiency: direct, indirect and total effect.



Model II – Spatial distribution of technical efficiency

Figure: II - Spatial distribution of technical efficiency: direct, indirect and total effect.