Bayesian MC^2 estimation of the capital-labor substitution elasticity in developing countries

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Abstract

A bayesian Markov Chain Monte Carlo estimation of the capital-labor substitution elasticity is proposed. This approach allows to incorporate economic theory, previous empirical work and expert criteria into the production function, thus overcoming the typical small-sample problems faced by developing countries during the estimation of parameters for their computable general equilibrium models.

JEL codes: C11, D2, D5
Keywords: Bayesian analysis, production function, computable general equilibrium models

1 Introduction

Policy-makers need to estimate long-run elasticities to calibrate their computable general equilibrium models[1], since a model with a realistic calibration strongly supports the policy formulation process. Among the parameters needed to calibrate these models, considerable effort has focused on the estimation of the elasticity of substitution between capital and labor. In some developing countries, the data to estimate this elasticity is inaccurate and has a small sample size. Together with the usual estimation problems of endogeneity, omitted variable bias, nonstationarity, serial correlation, and specification problems related to functional forms, the low-quality data further casts doubt on the frequentist estimation of this parameter[2].

This paper proposes a Bayesian Markov Chain Monte Carlo (MC^2) estimation of the capital-labor substitution elasticity. The Bayesian approach allows to incorporate economic theory, previous empirical work and expert criteria into the estimation process, thus avoiding the typical problems related to data deficiencies in developing countries.
Section 2 describes the MC^2 estimation procedure, section 3 illustrates the technique with data of the Bolivian economy. Section 4 discusses the results and concludes.

2 CES and Bayesian MC^2

A constant returns to scale production function with a constant elasticity of substitution (CES) between two factors, capital (k) and labor (l), implies a production function of the form (Arrow et al., 1961),

$$q = A\left[\rho k^{\frac{\sigma-1}{\sigma}} + (1-\rho)l^{\frac{\sigma-1}{\sigma}}\right],$$

where q is real output, A is a Hicks-neutral technological shifter, $\rho \in [0,1]$ is a distribution parameter and $\sigma \in [0,\infty)$ is the elasticity of substitution between capital and labor. Two first-order conditions emerged from profit maximization by firms in a competitive framework; the labor demand obtained from these conditions equals,

$$\frac{q}{l} = \left(\frac{w}{p}\right)^{\sigma} (\rho)^{-\sigma} A^{1-\sigma}.$$

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[1] The reference to policy-makers does not exclude the relevance of the estimation procedure for other users of computable general equilibrium models, as e.g. academics and researchers.

[2] For example, in Bolivia it was not possible to estimate the capital-labor substitution elasticities of the general equilibrium model MACEPES (*Modelo de Análisis de Choques Exógenos y de Protección Económica y Social*, see Canavire and Mariscal, 2010).

This equation can be log-linearized,

$$y = \theta_0 + \theta_1 x, \tag{1}$$

for $y = ln(q/l)$, $x = ln(w/p)$, w/p real wages and $\theta_0 = -\sigma ln\rho + (1-\sigma)lnA$, $\theta_1 := \sigma$ (as in Cicowiez, 2011). Adding an error term and time or cross-section subscripts, equation 1 becomes a linear regression model. The regression form of 1 is used for the empirical estimation of σ with e.g. ordinary least squares (OLS) or two stage least squares (2SLS). A different approach is to use Bayesian methods. With Bayesian methods, it is posible to incorporate previous empirical work and economic theory into the CES empirical model 1, together with the expert criteria of professionals in the field.

Let $X = [\mathbf{1}_n \ (x_1,...,x_t)']$, $\theta = [\theta_0 \ \theta_1]$, $y = (y_1,...,y_t)'$,

$$y \sim \mathcal{N}(X\theta, s^2 I_n),$$

with prior distributions,

$$\underline{\theta} \sim \mathcal{N}(\theta_0, B_0), \ \ s^2 \sim \mathcal{IG}(\alpha_0/2, \delta_0/2).$$

As a consequence of these assumptions,

$$\bar{\theta}|s^2, y \sim \mathcal{N}(\underline{\theta}, B_1),$$

where,

$$\begin{aligned} B_1 &= [s^{-2}X'X + B_0^{-1}]^{-1}, \\ \underline{\theta} &= B_1[s^{-2}X'y + B_0^{-1}\theta_0], \end{aligned}$$

and,

$$\begin{aligned} \alpha_1 &= \alpha_0 + n, \\ \delta_1 &= \delta_0 + (y - X\theta)'(y - X\theta). \end{aligned}$$

Since both conditional posterior distributions are standard, a Markov Chain Monte Carlo sampler can be used to find the posterior distribution of (θ, s^2):

(a) Let $s^{2(0)}$ be a starting value of s^2.

(b) At the gth iteration,

$$\begin{aligned} \theta^{(g)} &\sim \mathcal{N}(\bar{\theta}^{(g)}, B_1^{(g)}), \\ s^{2(g)} &\sim \mathcal{IG}(\alpha_1/2, \delta_1^{(g)}/2), \end{aligned}$$

where,

$$\begin{aligned} B_1^{(g)} &= [s^{-2(g-1)}X'X + B_0^{-1}]^{-1}, \\ \bar{\theta}^{(g)} &= B_1^{(g)}[s^{-2(g-1)}X'y + B_0^{-1}\theta_0], \\ \delta_1^{(g)} &= \delta_0 + (y - X\theta^{(g)})'(y - X\theta^{(g)}). \end{aligned}$$

With a repetition of the Gibbs sampling (b) until $g = \mathcal{B} + \mathcal{G}$ —where $\mathcal{B}$ is the burn-in sample and $\mathcal{G}$ is the desired sample size— is posible to calculate values of $\theta^{(g)}$ and $s^{2(g)}$, $g = \mathcal{B}+1,...,\mathcal{B}+\mathcal{G}$ and simulate the posterior distribution of θ and s^2. The Bayesian point estimator $\hat{\theta}$ is the value of θ that minimizes the expected value of a loss function $L(\hat{\theta}, \theta)$, where the expectation is taken over the posterior distribution of θ, $\pi(\theta|y)$,

$$\min_{\hat{\theta}} \mathbb{E}[L(\hat{\theta}, \theta)] = \min_{\hat{\theta}} \int L(\hat{\theta}, \theta)\pi(\theta|y)d\theta.$$

Under quadratic loss, $L(\hat{\theta}, \theta) := \left(\hat{\theta} - \theta\right)^2$,

$$\min_{\hat{\theta}} \mathbb{E}[L(\hat{\theta}, \theta)] = \min_{\hat{\theta}} \int \left(\hat{\theta} - \theta\right)^2 \pi(\theta|y)d\theta.$$

Differentiating with respect to $\hat{\theta}$ and setting the derivative equal to zero,

$$\begin{aligned} \hat{\theta} &= \int \theta\pi(\theta|y)d\theta \\ &= \mathbb{E}(\theta|y). \end{aligned}$$

i.e. the optimal point estimator under quadratic loss is the mean of the simulated posterior distribution of θ. See Geweke (2005) or Gill (2007).

Prior elicitation in the CES model

It is a common Bayesian practice to use maximum-likelihood or OLS estimates as priors for parameters like the variance or the covariance contained in B_0, since economic theory rarely provides values for these statistical concepts. Nevertheless, the CES model offers a unique opportunity to prior elicitation, as there is no need to use values from previous estimations which, given the low-quality data in developing countries, would be certainly doubtful.

Let $\underline{\theta}_1$ be a prior elicitation of θ_1 and $\underline{Var}(\theta_1)$ the elicitated prior variance of θ_1 (these priors can be based on previous empirical work, economic theory or expert criteria, given the fact that in developing countries the *knowledge* and the *experience* of experts in the field is a valuable source of information that can be exploited to improve the estimation of the elasticities).

From equation 1 it is clear that the prior for θ_0 would be,

$$\underline{\theta}_0 = -\underline{\theta}_1 ln\rho + (1 - \underline{\theta}_1)lnA. \tag{2}$$

The components of the prior variance-covariance matrix B_0,

$$B_0 = \begin{bmatrix} \underline{Var}(\theta_0) & \underline{Cov}(\theta_0, \theta_1) \\ \underline{Cov}(\theta_0, \theta_1) & \underline{Var}(\theta_1) \end{bmatrix},$$

can be elicitated given the fact that,

$$\begin{aligned}\underline{Var}(\theta_0) &:= Var(-\underline{\theta}_1 ln\rho + (1-\underline{\theta}_1)lnA)\\ &= [(ln\rho)^2 + (lnA)^2 + 2ln(\rho + A)]\underline{Var}(\theta_1)\end{aligned} \tag{3}$$

and,

$$\underline{Cov}(\theta_0, \theta_1) = \mathbb{E}(\underline{\theta}_0\underline{\theta}_1) - \mathbb{E}(\underline{\theta}_0)\mathbb{E}(\underline{\theta}_1)$$

Thus, $\underline{\theta}_0$, $\underline{\theta}_1$ and B_0 can be fully elicitated assigning values to the hyperparametes A and ρ, *i.e.* factor productivity and labor participation[3]. The elicitation of these hyperparameters can also be based on expert criteria, previous empirical work or economic theory, without requiring a previous frequentist estimation.

3 Results

This section illustrates the Bayesian MC^2 estimation of σ with data of the industrial sector in the Bolivian economy, and compares the Bayesian approach with a traditional frequentist estimation. Table 3 shows estimates for other activities.

Frequentist approach

Bolivia is a typical example of a developing country that lacks of accurate data and has only a short sample of observations from aggregate activities. To estimate equation 1, the information of real output from national accounts was divided between the number of people employed in each activity (a proxy of the labor factor). Real income was measured deflating nominal earned income with the implicit deflator of output. The labor and wage proxies came from the bolivian household survey. Since this survey is only available yearly from 1996 to 2006[4], with no data for 1998 and only one observation for the years 2003-2004, these rough measures make a total data sample of 9 observations.

Table 1 shows the OLS estimation of the capital-labor substitution elasticity for the industrial activities based on this data. Model $\mathcal{M}_1$ is equal to the regression form of equation 1, $\mathcal{M}_2$ includes a trend term as in Jabbar (2002), and $\mathcal{M}_3$ includes a first order lag of y_t, as in Tipper (2011). The first model has an extremely low R^2, equal to .0956, and the OLS estimator of θ_1 is .10, not statistically different from zero. Including a trend term τ_t in $\mathcal{M}_2$ improves the coefficient of determination to .4904, but now the point estimator of θ_1 is negative, a theoretically impossible value. Finally, the OLS estimation of a model ($\mathcal{M}_3$) with a lag of y_t produces an extremely low estimate of θ, equal to .075, again not statistically different from zero. These doubtful estimations illustrate the fact that developing countries need to incorporate other information into their estimation strategies in order to improve the reliability of their estimates. This could be achieved with Bayesian methods.

Bayesian estimation (I): prior elicitation

The prior elicitation of θ_1 was based on the previous empirical work of Clague (1969), Behrman (1972) and Boon (1973), who estimate σ for Perú, Chile and México, respectively. The theoretical work of Lucas (1990) and Erceg et al. (2006) was also taken into consideration, together with the expert criteria of Andersen (2003), who believes that in Bolivia the improvements in basic education makes unskilled workers more useful in the production process and, due to the specification of production functions, makes easier to replace the low physical capital by non-skilled labor, increasing the elasticity of substitution related to the demand for production factors in all the sectors of the economy. These considerations (table 2) lead to an average prior value of $\underline{\theta}_1 = .6217$, with a standard deviation of 0.1818 ($\underline{Var}(\theta_1) = .0331$).

With a industrial labor participation of 55 per cent $\rho = 0.55$ and a productivity factor of $A_t = 1.6$, equations 2 and 3 lead to a prior of $\underline{\theta}_0 = .2902$ and a variance $\underline{Var}(\theta_0) = .0697$. Finally, values of $\alpha_0 = 1\times 10^2$ and $\delta_0 = 1\times 10^3$ were chosen to ensure a fast chain convergence. The complete prior elicitation equals,

$$\begin{aligned}&\underline{\theta}_1 = .6217,\\ &\underline{\theta}_0 = .2902,\\ &B_0 = \begin{bmatrix} .0697 & .0117 \\ .0117 & .0331 \end{bmatrix},\\ &\alpha_0 = 1\times 10^2,\ \delta_0 = 1\times 10^3.\end{aligned}$$

[3]The social accounting matrix can be a valuable source of information to elicitate these concepts. Operating on the first order conditions of the CES function (through the optimization problem that a firm solves under perfect competition),

$$\rho = \frac{wl^{\sigma^{-1}} + rk^{\sigma^{-1}}}{wl^{\sigma^{-1}}},$$

for r the capital return. Then, only when $\sigma = 1$, ρ strictly measures labor participation. Assuming $\sigma \approx 1$ facilitates the conceptual elicitation of ρ; in other circumstances, prior knowledge about capital stock and capital return will be required to elicitate ρ.

[4]The last available survey is from 2009, but the cut year chosen for the MAMS model is 2006.

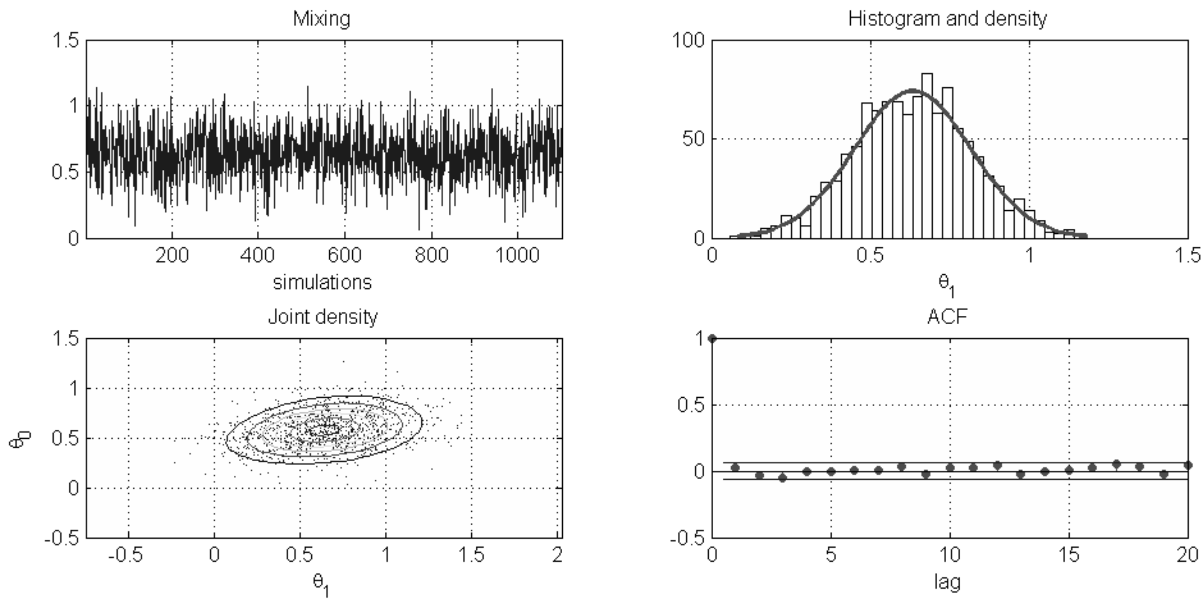

Figure 1: Simulations of the first MC[2] chain

Table 1: Frequentist OLS estimation[a]

	$\mathcal{M}_1$	$\mathcal{M}_2$	$\mathcal{M}_3$
θ_0	2.07	2.20	1.50
	(.0000)	(.0000)	(.0352)
θ_1	.10	–.03	.075
	(.3279)	(.7074)	(.4171)
R^2	.0956	.4904	.2034

[a] p-values between parentheses.
$\mathcal{M}_1$: $y_t = \theta_0 + \theta_1 x_t$
$\mathcal{M}_2$: $y_t = \theta_0 + \theta_1 x_t + \theta_2 \tau_t$
$\mathcal{M}_3$: $y_t = \theta_0 + \theta_1 x_t + \theta_3 y_{t-1}$

Bayesian estimation (II): MC^2 results

Based on the previous prior elicitation, two parallel chains of size $g = 1100$ were run, discarding the first 100 simulations (the burn-in sample $\mathcal{B}$). Figure 1 shows a good mixing of the first chain[5]. The ellipsoidal shape of the joint posterior density between the simulations of θ_0 and θ_1 emerges from the positive prior covariance in B_0. No evidence of autocorrelation is visible in the sampling autocorrelation functions (ACF) of the chain. The Gelman-Rubin statistic (Gelman and Rubin, 1992) is approximately equal to one (table 3), indicating that both chains converge to the same posterior distribution. The density of the simulated distribution is symmetric with a support above zero, suggesting a positive elasticity of substitution between capital and labor, as expected from economic theory. The Bayesian point estimator of this elasticity is equal to $\hat{\theta}_1 = .63$, with a Bayesian credible interval at a 95 level equal to [.26,.99]. The amplitude of the credible interval indicates gross capital-labor complementarity in the Bolivian industry, suggesting that the production factors in this sector are not fixed (the case of $\sigma = 0$) but they are not perfect substitutes neither (the case of $\sigma \to \infty$). At a 99 level, the bayesian credible interval of θ_1 is equal to [.16,1.09]. Thus, the intervalic results strongly reject the existence of a Leontief production function in the Bolivian industry, but the rejection of a Cobb-Douglas function is marginal at a 95 level and cannot be rejected at a 99

[5]The results of the second chain are available upon request.

Table 2: Information for the prior elicitation of θ_1

Previous studies[a]		Theoretical work[b]		Expert criteria[c]	
Behrman	.76	Lucas	.60	Andersen	.80
Boon	.74	Erceg et al.	.50		
Clague	.33				

[a] Estimations of σ for Chile, México and Perú. Chile: Jere Behrman, *Elasticidades de sustitución sectoriales entre capital y trabajo en una economía en vías de desarrolllo: análisis de series de tiempo para el periodo de post-guerra en Chile*. México: Gerard K. Boon, *Sustitución de capital y trabajo, comparaciones de productividad e insumos primarios proyectados*. Perú: Christopher Clague, *Capital-labor substitution in manufacturing in underdeveloped countries*.

[b] Robert E. Lucas Jr., *Supply-Side Economics: An Analytical Review*. Christopher J. Erceg, Luca Guerrieri, Christopher Gust, *SIGMA: a new open economy model for policy analysis*.

[c] Likke Andersen, *Educación en Bolivia: El efecto sobre el crecimiento, el empleo, la desigualdad y la pobreza.*

Table 3: Bayesian estimation of θ_1

	$\hat{\theta}_1$	Bayesian CI	Gelman-Rubin
Industry	.6339	[.26, .99]	.99956
Agriculture	.1713	[.008,.39]	.99961
Extractive activities[a]	.5144	[.08, .94]	.99955
Electricity, gas, water	1.1061	[.39,1.80]	.99955
Construction	.1944	[.01, .38]	.99961
Trade	.3416	[–.33,.97]	.99956
Transportation	.2641	[.002,.53]	.99961
Services[b]	.5341	[–.03, 1.11]	.99958

[a] Crude oil, natural gas and mining

[b] Includes financial services

level. Then, the existence of this type of production function cannot be excluded completely for the Bolivian industry. Table 3 shows the estimation results for other economic activities, with $\hat{\theta}_1$ estimated following a similar MC^2 approach.

4 Discussion

The estimation of σ in developed countries is close to one or even greater than one; see *inter alia* Arrow (1961), Maddala and Kedane (1966), Feldstein (1967), Burras and Moroney (1975) or, more recently, Antras (2004). On the contrary, the estimated value of σ in this study is congruent with the empirical fact that in developing countries the estimation of σ tends to be lower (Antony, 2009), as in Clague (1969), Behrman (1972), Boon (1973), Jabbar (2002) or Cicowiez (2011). Thus, the Bayesian approach seems an interesting alternative for the estimation of the capital-labor substitution elasticity in developing countries, as with this technique it is posible to compensate the data limitations with the inclusion of the experience of professionals in the field, theoretical concepts or previous empirical work, trough prior elicitation. The combination of this prior criteria with data evidence produces an estimator that exploits both sources of information. Compared with traditional frequentist estimations, the Bayesian estimator of the capital-labor substitution elasticity is theory-consistent, and thus can be used to properly calibrate computable general equilibrium models.

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